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Efficiency Analysis of Indian Private Sector Banks in a Developing Economy – A Data Envelopment Analysis Approach

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Abstract

One crucial stage in the production process is evaluating performance. Measuring production efficiency of decision making units is a vital issue for economic theorists and economic policy makers. Though there are many tools available for measuring productivity efficiency, since 1978 Data Envelopment Analysis has been emerging as a new management science for the performance analysis of production units and a very powerful tool to measure the technical efficiency of the decision making units. Data Envelopment Analysis, a non-parametric methodology based on a linear programming approach provides efficiency scores for similar organizations, known as Decision Making Units (DMUs), with many inputs and outputs, and benchmarks inefficient units against peers. DEA can be described as data-oriented as it effects performance evaluations and other inferences directly from the observed data with minimal assumption. DEA evaluates efficiency of decreasing units relative to all other units with the simple restriction that all DMUs are on or below the extreme frontier. The DMUs are then ranked according to the quantity of peers. Nonetheless, ties across ranks can be settled by Andersen-Peterson's model. Applying Seiford & Zhu's requirements for infeasibility identification validates the infeasible conclusion that the super-efficiency model yields. Traditional DEA models evaluate the performance or efficiency of a particular DMU relative to other DMUs in the set including the evaluated DMU. In contrast, when a DMU under evaluation is not included in the reference set of the envelopment model, the resulting DEA models are called super-efficiency DEA Models. The findings of our empirical study, which focused on three inputs and two outputs, of Private Sector Banks in India in 2019–20 is provided. Banker, Charnes, and Cooper (BCC) model and super-efficiency models based on the variable return to scale (VRS) assumption were employed. Sensitivity analysis was also performed and the infeasibility issue was discovered and a stability region was also created.

Keywords: Data Envelopment Analysis, Private Sector Banks, Decision Making Units, BCC Model, Super-Efficiency, Infeasibility, Sensitivity Analysis, Stability Region.

Introduction

Banking is an essential part of the entire financial system. The banking industry supports economies throughout the world. The banking sector has a significant impact on the growth and development of any country's economy. India's banking sector has been essential to the country's economy, which is among the fastest-growing in the world. The Indian banking sector has been greatly impacted by recent technological developments, an increase in international investment, and regulatory adjustments.

Following a protracted and difficult past, India's banking sector is now an essential part of the nation's economy. With the adoption of new technologies, the introduction of state-of-the-art products and services, and the expansion of its reach to previously marginalised segments in society, the sector have experienced a considerable transition in recent years. Despite the many obstacles the industry must overcome, it is well-positioned to do so and grow even more. The banking industry is predicted to have a big impact on India's future economic growth and prosperity. In this industry, banks come in a wide variety of forms, including foreign, cooperative, private, and public sector banks.

Private sector banks are those in which a private firm or a group of people owns the majority of the bank's equity. They have a distinct financial system but nonetheless abide by the central bank's rules. The purpose of this article is to assess the performance of Private Sector Banks in India through the use of Data Envelopment Analysis. Data Envelopment Analysis is a non-parametric technique that is used to compare the relative effectiveness of comparable organizations, often known as Decision Making Units (DMUs). This mathematical programming technique, which is based on a linear programming issue, is a useful tool for determining how well decision-making units are working. Each decision-making unit produces the same outputs using the same inputs. By examining DMUs with many inputs and numerous outputs, DEA calculates the efficiency frontier.

Technical, scale, allocative, economical, and other types of efficiency are possible. DEA has been used by numerous researchers in the past few years to solve various real-world challenges. Numerous sectors, including business, finance, healthcare, education, and agriculture, are among those in which DEA is used in real life. Consequently, both the service and non-service sectors have found use for DEA.

Review of literature

Despite the fact that there are many other techniques for measuring productivity efficiency, Data Envelopment Analysis has emerged as a new management science for the performance analysis of production units and a very powerful tool to measure the technical efficiency of decision-making units since 1978. The extension of Operations Research, which is predicated on optimisation techniques for resource allocation issues, is Data Envelopment Analysis. The area of Data Envelopment Analysis has produced over 7000 research articles and a sizable number of books since 1957.

The concept of technical efficiency of input based on disposability criterion states that "the vector of inputs is technically efficient if and only if increasing any output or reducing any input is achievable only by increasing some other input or decreasing some other output" (Koopman, 1951). Radial measure is a word used to describe a technical efficiency measure expressed in terms of the highest proportionate expansion of all output or decrease of all variable inputs that is conceivable (Debreu, 1951).
There has been a significant advancement in the field of measuring productive efficiency. Development shifted to concentrate on the several efficiencies referred to as "scale," "allocative," and "technical" (Farrel, 1957). Thus, the series of technical efficiency laid the groundwork for the study of frontier analysis. (Farrel, 1957; Debréu, 1951).

A linear fractional programming technique with a mathematical foundation was developed and the first thorough analysis of DEA was completed (Charnes and Cooper, 1962). In the case of multiple outputs, an efficient linear programming-based computer process based on Farrel's ideas was created for a variety of Technical Efficiency issues (Bressler, 1966; Boles, 1966; Seitz, 1966; Sitorus, 1966).

The production efficiency metric, which Farrel initially presented in 1957, was expanded upon by the development of the basic DEA model (Charnes, Cooper, and Rhodes, 1978). This model, called the CCR Model, is predicated on the Constant Returns to Scale assumption (CRS). The recommended model (CCR Model) provides an efficiency measure of each decision making unit in the sample that maximises the ratio of weighted outputs to weighted inputs, subject to the limitations that the similar ratio must be less than or equal to unity for all DMUs.

The Variable Returns to Scale assumption (VRS) is supported by the expanded CCR Model (Banker, Charnes, and Cooper, 1984). The addition of a new independent variable by Banker et al. allows one to determine whether operations are conducted in regions of expanding, constant, or diminishing returns to scale in this model. This model provides estimates of the technical and scale efficiency of each sample decision-making unit with respect to the efficiency frontier.

A parametric method-based methodology for assessing effective units and enabling comparison in Data Envelopment Analysis was developed (Anderson and Peterson, 1993). The use of super-efficiency in DEA sensitivity analysis was proposed (Zhu, 2001). Additionally, this work establishes the sufficient and necessary requirements to maintain the same efficiency classification even when data variation is applied to all DMUs.

Research was carried out to evaluate the efficiency of municipal water supply systems using Data Envelopment Analysis (AbuSerriya, Hamed and Agha, 2023). The use of Data Envelopment Analysis is illustrated by evaluating the relative financial strength of thirteen financial firms by benchmarking the financial ratios of a firm against its peers (Malhotra, Malhotra and Lafond, 2009).

An approach grounded on DEA was employed to assess and prioritise the achievements of Academy Award winners for Best Original Score between 1990 and 2016 (Asadpour and Shirouyehzad, 2019). In the banking sector, Data Envelopment Analysis was utilised to evaluate the operational efficacy of 14 saving bank branches (Sherman and Gold, 1985). The performance of the banking sector in the US and a few European countries was also investigated using Data Envelopment Analysis (Pastor, Perez, and Quesada, 1997). The efficiency of 750 selected European banks was examined using an intermediation approach (Casu and Molyneux, 2003). The efficiency of the Turkish banking sector was assessed using an interval DEA approach (Budak and Erpolat, 2013) and the results showed that interval DEA methods yield more selective results than DEA.

Efficiency estimates for Indian public sector banks' costs, revenues, and profits in 2015–2016 were evaluated using the VRS assumption (Jayarani and Prakash, 2017). Using Data Envelopment Analysis, another study

(Khurana and Khosla, 2019) looks at technical efficiency and it correlates with regard to the banking sector in India between 1995 and 2016. 94 public, private, foreign, and small financial commercial banks in India were evaluated for technical efficiency in 2019 using Data Envelopment Analysis (DEA), and the Tobit regression method was used to assess the factors that influence technical efficiency (Lakshmanasamy, 2021).

Objective of the study

After the liberalization in 1991, a large no. of private sector banks came into existence. Currently, there are 22 private sector banks. Unlike public sector banks, these banks are not owned by the government but by individuals or private corporations. According to the consolidated balance sheet of scheduled banks, 32.8 percent of the total assets of the scheduled commercial banks pertain to private sector banks as of March 31, 2021. The above facts and figures motivated the author to carry out the performance analysis of private sector banks in India.

In this article, an attempt is made to find the efficiency evaluation of Private Sector Banks in India for the period 2019-20 through basic models of DEA. Sensitivity Analysis also has been carried out followed by construction of stability region.

Methodology

Banker Charnes and Cooper (BCC) Model

Banker, Charnes and Cooper (1984) proposed a model with convexity constraint which permits variable return to scale (VRS) assumption. This model is generally known as input oriented BCC DEA Model and is presented below.

Assume that there are n units each consuming m inputs to produce s outputs. Let y_{rj} denote the level of the rth output (r = 1,2,…,s) from unit j (j = 1,2,…,n) and x_{ij} denotes the level of the ith input (i = 1,2,…,m) to the jth unit.

Model I : BCC Input Oriented DEA Model

$\theta^* = \min \theta$

Subject to

$$\sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta x_{io} \text{ for } i = 1,2, \dots m$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro} \text{ for } r = 1,2, \dots s$$

$$\sum_{j=1}^{n} \lambda_j = 1$$

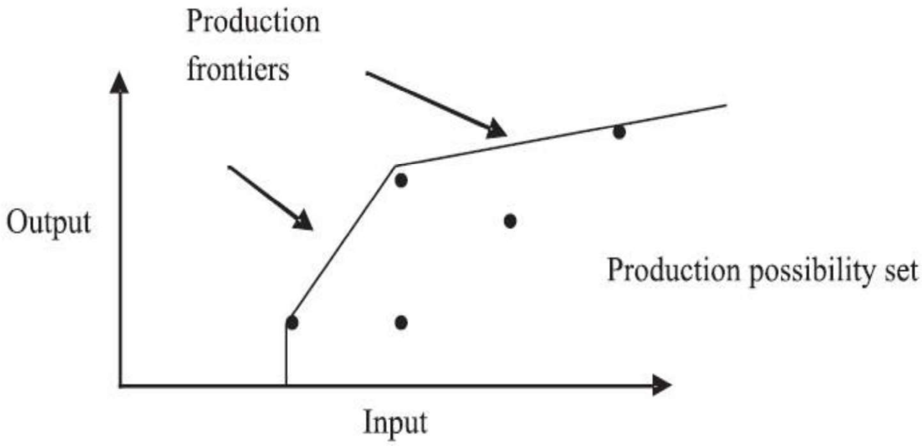
$$\lambda_j \geq 0 \text{ for } j = 1,2, \dots n$$

This model is termed as envelopment version and it obtains efficiency score (θ^*) and weights (λ) for each DMU. The variable θ, which appears in the objective function, represent the amount of proportional reduction applied to all inputs of evaluated DMU to improve its efficiency. It may be noted that the simultaneous reduction of all inputs results in a radial movement towards the envelopment surface. The evaluated DMU (DMU_o) is said to be efficient if

(i) $\theta^* = 1, \lambda_j = 1$ for $j = o$ and $\lambda_j = 0$ for $j \neq o$

(ii) All slacks are zero.

If $0 < \theta^* < 1$, it indicates inefficiency. BCC Model is similar to the CCR Model except for the convexity constraint $\sum_{j=1}^{n} \lambda_j = 1$. This convexity constraint ensures the piecewise linear approximation to the efficiency frontier and concave characteristics which leads to VRS characterization. These aspects are pictorially represented as follows



Production Frontier of the BCC Model

The above model is a primal one and its dual known as BCC output maximization is given below.

Model II : BCC Output Maximization Model

Here, Banker et al (1984) introduced a new separate variable (u_0) in the objective function which admits VRS characterization. This variable u_0 identifies whether the operations were conducted in the region of increasing, constant or decreasing return to scale in case of multiple input and multiple output scenario.

$$\max_{u,v} Z_o(u,v) = \sum_{r=1}^{s} u_r y_{ro} + u_0$$

Subject to

$$\sum_{i=1}^{m} v_i x_{io} = 1$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + u_0 \leq 0 \text{ for } j = 1,2, \dots n$$

$$v_i \geq \varepsilon \text{ for } i = 1,2, \dots m$$

$$u_r \geq \varepsilon \text{ for } r = 1,2, \dots s$$

u_0 free in sign

The optimal solution of this model (Z_o^*, $u_r$and v_i) represents the efficiency scores, weights of outputs and inputs respectively for the evaluated DMU_o. DMU_o is efficient if and only if $Z_o^* = \max Z_o = 1$, otherwise it is inefficient. Next, BCC developed output oriented DEA Model (envelopment version) and the same is given below.

Model III : BCC Output Oriented DEA Model

$$\phi^* = \max \phi$$

Subject to

$$\sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{io} \text{ for } i = 1,2, \dots m$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \geq \phi y_{ro} \text{ for } r = 1,2, \ldots s$$

$$\sum_{j=1}^{n} \lambda_j = 1$$

$$\lambda_j \geq 0 \text{ for } j = 1,2, \ldots n$$

This model results in efficiency score (ϕ^*) and weights (λ) for each DMU. The above model maximizes on ϕ to achieve proportional output augmentation with $\phi = 1$ indicating efficiency and $\phi > 1$ indicating an extent of radial inefficiency related to the best practice DMU included in the sample.

Super-Efficiency Models

It is well established that the fundamental DEA methodology—CCR and BCC—is an extremely effective instrument for evaluating the effectiveness or performance of comparable organizations. However, DEA displays many units as efficient in the situation of a high number of inputs and outputs because of the restriction on the efficiency score that $\phi^* = 1$ in output oriented. As a result, there may be ties in rankings among the DMUs in the sample taken into consideration for the study when we attempt to rate the DMUs based on the peer summary. The idea of super-efficiency was presented in order to tackle this problem. Conventional DEA models assess a DMU's effectiveness or performance in comparison to other DMUs in the set that includes the DMU under evaluation. On the other hand, the resultant DEA models are referred to as super-efficiency DEA Models when the DMU being evaluated is not part of the envelopment model's reference set.

VRS Super-Efficiency DEA Models

Seiford and Zhu (2002) proposed VRS SE DEA model based on Anderson and Peterson (1993) SE DEA model in respect of input oriented and output oriented and are presented below.

Model IV : VRS SE DEA Model (Input Oriented)

$$\min \theta^{SE}$$

Subject to

$$\sum_{\substack{j=1 \\ j \neq o}}^{n} \lambda_j x_{ij} \leq \theta^{SE} x_{io} \text{ for } i = 1,2, \ldots m$$

$$\sum_{\substack{j=1 \\ j \neq o}}^{n} \lambda_j y_{rj} \geq y_{ro} \text{ for } r = 1,2, \ldots s$$

$$\sum_{\substack{j=1 \\ j \neq o}}^{n} \lambda_j = 1$$

$$\lambda_j \geq 0 \text{ for } j \neq o$$

This model yields SE Scores of the evaluated DMU and weights to all other DMUs included in the set. The SE Scores categorize (Charnes, Cooper and Thrall, 1991) the set of DMUs into four groups namely.

(i) E: set of extreme efficient DMUs ($\theta^{SE*} > 1$)

(ii) E′: set of efficient DMUs that are not extreme points ($\theta^{SE*} = 1$)

(iii) F: set of weakly efficient DMUs ($\theta^{SE*} = 1$ but with non − zero slacks)

(iv) N: set of inefficient DMU ($\theta^{SE*} < 1$)

The new facet could be formed by the reference set (which may include F and N) corresponding to each extreme efficient DMU in the set E. This new facet represents min input combination used to produce the same level of output

Model V : VRS SE DEA Model (Output Oriented)

$\max \phi^{SE}$

Subject to

$$\sum_{\substack{j=1 \\ j \neq o}}^{n} \lambda_j x_{ij} \leq x_{io} \text{ for } i = 1,2, \dots m$$

$$\sum_{\substack{j=1 \\ j \neq o}}^{n} \lambda_j y_{rj} \geq \phi^{SE} y_{ro} \text{ for } r = 1,2, \dots s$$

$$\sum_{\substack{j=1 \\ j \neq o}}^{n} \lambda_j = 1$$

$\lambda_j \geq 0$ for $j \neq o$

The solution of the model yields SE Scores of the evaluated DMU and weights to all other DMUs in the sample. The SE Scores categorize (Charnes, Cooper and Thrall, 1991) the set of DMUs into four groups namely.

(i) E: set of extreme efficient DMUs ($\theta^{SE*} > 1$)

(ii) E′: set of efficient DMUs that are not extreme points ($\theta^{SE*} = 1$)

(iii) F: set of weakly efficient DMUs($\theta^{SE*} = 1$ but with non − zero slacks)

(iv) N: set of inefficient DMU ($\theta^{SE*} < 1$)

The new facet formed by the reference set represent that no other DMU produces the max output with the given input combinations. It may be noted that the frontier line will not alter if the inefficient and weak efficient DMUs are excluded from the new facet.

Problem of Infeasibility

It is known that the evaluated DMU is excluded from the reference set in the SE DEA Models and it causes the problem of infeasibility. A necessary and sufficient condition says that infeasibility occurs in the case of extreme efficient DMUs. Also, it may be noted that extreme efficient DMUs have either feasible or infeasible solution. Infeasibility may arise in CRS SE input oriented DEA Model with the necessary and sufficient condition in case of occurrence of certain patterns of zero data in inputs and outputs (Zhu, 1996). But CRS SE output oriented DEA Model does not encounter the problem of infeasibility. The problem of infeasibility mainly arises in VRS SE DEA Model, because of its scale assumption (VRS). Chen and Liang (2011) stated the following theorem which explains the sufficient condition for infeasibility in VRS SE DEA Model.

Theorem 1: Statement

A sufficient but not necessary condition for infeasibility of the **input-oriented** VRS SE models is that there exists at least one output that has a value for the evaluated DMU_o greater than the values of any other DMUs.

Theorem 2: Statement

A sufficient but not necessary condition for infeasibility of the **output-oriented** VRS SE models is that there exists at least one input that has a value for the evaluated DMU_o greater than the values of any other DMUs.

It may be noted that efficient DMU under standard BCC DEA model is either extreme efficient or infeasible in VRS SE Model. Lee, Chu and Zhu (2011) proposed the following theorems for the identification of infeasibility that arises in input oriented and output oriented models

Theorem 3: Statement

The input-oriented super-efficiency VRS model is infeasible if and only if $g^* < 1$ where $g^*$is the optimal value to

$$g^* = \max g$$

Subject to

$$\sum_{\substack{j=1 \\ j\neq o}}^{n} \lambda_j y_{rj} \geq g y_{ro} \text{ for } r = 1,2,\ldots s$$

$$\sum_{\substack{j=1 \\ j\neq o}}^{n} \lambda_j = 1$$

$$\lambda_j \geq 0 \text{ for } j \neq o$$

Theorem 4: Statement

The output-oriented super-efficiency VRS model is infeasible if and only if $h^* > 1$ where h^* is the optimal value to

$$h^* = \min h$$

Subject to

$$\sum_{\substack{j=1 \\ j\neq o}}^{n} \lambda_j x_{ij} \leq h x_{io} \text{ for } i = 1,2,\ldots m$$

$$\sum_{\substack{j=1 \\ j\neq o}}^{n} \lambda_j = 1$$

$$\lambda_j \geq 0 \text{ for } j \neq o$$

Efficiency Sensitivity Analysis

Sensitivity analysis plays a vital role in the performance evaluation through Data Envelopment Analysis. In recent years, many researchers have studied efficiency sensitivity to perturbations in the data which is an important issue in DEA. Initially Charnes, Cooper, Golany, Seiford and Stutz (1985) studied sensitivity analysis that examines the change in single output. Further, Ahn and Seiford (1993) focussed on sensitivity of DEA results to the variable and model selection. Besides, some studies on sensitivity analysis were developed for the determination of range within which the variation is possible for any DMU. Degrees of freedom approach, algorithmic approach, metric approach and multiplier model approach are some more avenues for studying sensitivity analysis in DEA. It may be noted that super-efficiency DEA is another approach for studying DEA Sensitivity Analysis and it allows perturbations in data to preserve efficiency classification in the following cases.

(i) either in any input or any output in test DMU.

(ii) either in any input or any output in test DMU and remaining DMUs

(iii) any specific inputs and outputs in test DMU and remaining DMUs.

In this study, the author attempted DEA sensitivity analysis based on SE DEA model (Zhu 1996), in the case of data perturbations to output only in test DMU.

It is obvious that increasing any output or decreasing any input in the test DMU, the application of sensitivity analysis to perturbations in the data does not worsen the efficiency classification. So sensitivity analysis may be suggested to perform in the situation where decreasing any output or increasing any input alter the efficiency classification. Below are the forms, which may be considered for proportional increase of inputs or proportional decrease of outputs.

$$\hat{x}_{io} = \beta_i x_{io}; \qquad \beta_i \geq 1, i = 1,2, \dots m \qquad (3.1)$$

$$\hat{y}_{ro} = \alpha_r y_{ro}; \qquad 0 < \alpha_r \leq 1, r = 1,2, \dots s \qquad (3.2)$$

where x_{io} $(i = 1,2, \dots m)$ and y_{ro} $(r = 1,2, \dots s)$ are the inputs and outputs of the specific extreme efficient DMU_o respectively. Zhu (2003) stated that any increase of input or decrease of output will definitely change the status of DMUs in the efficient set E' to the inefficient set N. The DMUs in the weak efficient set F retain their same status irrespective of the amount of inputs increased or the amount of output decreased. So the sensitivity issues in the set E or F can be ignored. Thus, only the set of extreme efficient DMUs (E) would be subjected for sensitivity analsyis. In this context stability region could be constructed for every extreme efficient DMU to maintain its efficiency status.

kth Specific DEA Models

Seiford and Zhu (1998) suggested VRS kth specific DEA models in both orientation to construct the stability region for the extreme efficient DMUs and the same is listed below.

Model VI : VRS kth Specific Output Oriented DEA Model

$\max \alpha_k^o$ for each $k = 1,2, \dots s$

Subject to

$$\sum_{\substack{j=1 \\ j \neq o}}^{n} \lambda_j y_{rj} \geq \alpha_k^o y_{ro}$$

$$\sum_{\substack{j=1 \\ j \neq o}}^{n} \lambda_j y_{rj} \geq y_{ro} \text{ for } r \neq k$$

$$\sum_{\substack{j=1 \\ j \neq o}}^{n} \lambda_j x_{ij} \leq x_{io} \text{ for } i = 1,2, \dots m$$

$$\sum_{\substack{j=1 \\ j \neq o}}^{n} \lambda_j = 1$$

$\lambda_j \geq 0$ for $j \neq o$

For the above model, x_{ij} $(i = 1,2, \dots m)$ and y_{rj} $(r = 1,2, \dots s)$ represent the i^{th} input and r^{th} output of DMU_j $(j = 1,2, \dots n)$ respectively. Here the optimal values $(\alpha_k^*,\ k = 1,2, \dots s)$ of the model provide hypothetical frontier points. This frontier points from the output stability region that allows possible output decrease in each individual output for the observed DMU_o. Thus DMU_o could preserve its status in the same efficiency classification keeping all other outputs constant. Zhu (2003) define the following output stability region.

Definition

A region of allowable output decrease is called an Output Stability Region (OSR) if and only if DMU_o remains efficient after such decreases occur.

By solving the above models for each extreme efficient DMU, the solution obtained may be feasible or infeasible. Output stability region is of the form $\alpha_k^o < \alpha_k \leq 1$ in case of feasibility and it allows maximum α_k^o times of decrease in specific kth output from current level to preserve same efficiency classification. α_k, the adjusted factor, is an arbitrary value and it lies within the output stability region ($\alpha_k^o < \alpha_k \leq 1$). In the case of infeasibility, the output stability region is of the form $\alpha_k \leq 1$. The same efficiency classification can also be preserved by any value lesser than or equal to one multiplied with the specific kth output. Thus the OSR is of the closed form in case of feasibility and has no lower limit in case of infeasibility for data variation.

Zhu (2005) explains infeasibility and stability for the kth specific model through the following theorems.

Theorem 6: Statement

For an efficient DMU_o, a decrease of the kth output only, Model VI is infeasible, if and only if the amount of kth output of DMU_o can be decreased without limitation while maintaining the efficiency of DMU_o.

The above discussions clearly explain the application of the data variation to the test DMU for a specific output.

Data structure

The Department of Research and Statistics, Indian Banks' Association, Mumbai, published reports on the Performance Highlights of Private Sector Banks 2019–20. These reports contain the data that was taken into consideration for analysis. There were 22 private sector banks in India in 2019–20, and each one is taken into account for the performance analysis when referring to the 2019–20 research period. Every bank is seen as a decision-making unit. Here, the author takes into account five input and three output variables, all of which have been carefully chosen based on earlier research on banking performance.

BCC output-oriented DEA Models are used in this study to estimate the technical efficiency in order to evaluate the performance of Private Sector Banks operating in India. After carefully reviewing a number of previous studies pertaining to the performance in the national and international banking sectors, the author chose the input and output variables for the current investigation. According to the author, the production process greatly benefits from the careful selection of variables in this study, including input and output. Moreover, efforts are undertaken to verify and consistently test the data points. The Private Sector Banks (DMUs) considered for this study are listed in the following table.

Table 1: List of Private Sector Banks

S.No.	Name of the Bank	DMU	S.No.	Name of the Bank	DMU
1.	City Union Bank Ltd.	DMU 1	12.	The South Indian Bank Ltd.	DMU 12
2.	Tamilnad Mercantile Bank Ltd.	DMU 2	13.	Axis Bank Ltd.	DMU 13
3.	The Catholic Syrian Bank Ltd.	DMU 3	14.	DCB Bank Ltd.	DMU 14
4.	Dhanlaxmi Bank Ltd.	DMU 4	15.	HDFC Bank Ltd.	DMU 15
5.	The Federal Bank Ltd.	DMU 5	16.	ICICI Bank Ltd.	DMU 16
6.	The Jammu & Kashmir Bank Ltd.	DMU 6	17.	Indusind Bank Ltd.	DMU 17
7.	The Karnataka Bank Ltd.	DMU 7	18.	Kotak Mahindra Bank Ltd.	DMU 18
8.	The Karur Vysya Bank Ltd.	DMU 8	19.	Yes Bank Ltd.	DMU 19
9.	The Lakshmi Vilas Bank Ltd.	DMU 9	20.	Bandhan Bank	DMU 20
10.	The Nainital Bank Ltd.	DMU 10	21.	IDFC First Bank	DMU 21
11.	RBL Bank	DMU 11	22.	IDBI Bank Ltd.	DMU 22

The input and output variables selected in this study for the Private Sector Banks is listed below.

Table 2: Input Variables

S.No.	Input Variables	Units	Notation
1.	Deposits	Amount in Crores	IP1
2.	Fixed Assets	Amount in Crores	IP2
3.	Operating Expenses	Amount in Crores	IP3
4.	Staff	Number of Persons	IP4
5.	Paid-up capital	Amount in Crores	IP5

Table 3: Output Variables

S.No.	Output Variables	Units	Notation
1.	Total Income	Amount in Crores	OP1
2.	Investments	Amount in Crores	OP2
3.	Loans & Advances	Amount in Crores	OP3

This makes up the data matrix, which serves as the study's foundation. Three output variables and five input variables make up the data structure used in this investigation. A few statistical measures, including Mean, Minimum, Maximum, Range, and Standard Deviation of various input and output variables for Private Sector Banks, are computed and displayed in the following table in order to provide an overview of the features of this data set.

Table 4: Descriptive Statistics

	N	Range	Minimum	Maximum	Mean	Std. Deviation
Deposits	22	1139822.86	7679.43	1147502.29	189047.4341	290243.41278
Fixed Assets	22	8369.01	41.28	8410.29	1738.3014	2428.54903
Operating Expenses	22	30535.10	162.43	30697.53	5741.8073	7837.33361
Staff	22	116000.00	971.00	116971.00	25246.5909	32353.88556
Paid-up capital	22	10309.23	71.36	10380.59	1221.1823	2317.51300
Total Income	22	137345.78	727.69	138073.47	24820.0518	34837.22436
Investments	22	390150.44	1676.22	391826.66	58774.1441	94603.08749
Loans & Advances	22	989873.90	3828.98	993702.88	164726.4636	250485.82986

In the process of understanding this data set better, the author is interested in further examining the relationship between input and output variables. In view of this, the author attempted correlation analysis between the input and output variables and the results are outlined below.

Table 5: Correlation Analysis

Input Variables		**Output Variables**		
		Total Income	**Investments**	**Loans & Advances**
Deposits	Pearson Correlation	0.984	0.989	0.995
	Sig. (2-tailed)	0.000	0.000	0.000
	N	22	22	22
Fixed Assets	Pearson Correlation	0.679	0.701	0.662
	Sig. (2-tailed)	0.001	0.000	0.001
	N	22	22	22
Operating Expenses	Pearson Correlation	0.996	0.984	0.993
	Sig. (2-tailed)	0.000	0.000	0.000
	N	22	22	22
Staff	Pearson Correlation	0.962	0.944	0.963
	Sig. (2-tailed)	0.000	0.000	0.000
	N	22	22	22
Paid-up capital	Pearson Correlation	0.074	0.097	0.015
	Sig. (2-tailed)	0.742	0.668	0.948
	N	22	22	22

From Table 5, it may be observed that the input variable considered, except for Paid-up Capital, have a significant relationship with the selected output variable. Though one input variable does not show significant relationship with the output variables, Paid-up Capital is practically considered as an important input variable in Banking Sectors and plays a vital role in Data Envelopment Analysis and has thus been considered for the study.

Empirical investigation

BCC – Efficiency Scores and Peer Weights

The application of BCC output oriented Model to Private Sector Bank data yielded the following results.

Table 6: Efficiency Scores and Peer Weights – BCC Model

DMU	Efficiency Score	Reference DMU Weights				
DMU 1	1	λ_1 =1				
DMU 2	1	λ_2 =1				
DMU 3	1	λ_3 =1				
DMU 4	1	λ_4=1				
DMU 5	1	λ_5 =1				
DMU 6	1	λ_6=1				
DMU 7	1.0702	λ_1= 0.0794	λ_{10}=0.1225	λ_{12}=0.7386	λ_{19}=0.0595	
DMU 8	1.0807	λ_1 =0.4989	λ_{10}=0.4106	λ_{15} =0.0200	λ_{17}=0.0532	λ_{19}=0.0173
DMU 9	1.4077	λ_3 =0.1191	λ_4=0.7314	λ_{10} =0.0964	λ_5=0.0050	λ_{19}=0.0481
DMU 10	1	λ_{10} =1				
DMU 11	1	λ_{11} =1				
DMU 12	1	λ_{12} =1				
DMU 13	1	λ_{13}=1				
DMU 14	1.0816	λ_1 =0.0429	λ_{10}=0.6921	λ_9=0.2068	λ_{19}=0.0582	
DMU 15	1	λ_{15}=1				
DMU 16	1.0642	λ_4 =0.0983	λ_{15}=0.6483	λ_{19}=0.2346	λ_{21}=0.0188	
DMU 17	1	λ_{17}=1				
DMU 18	1.2103	λ_5 =0.0453	λ_{12}=0.2888	λ_{15}=0.1552	λ_{19}=0.5107	
DMU 19	1	λ_{19}=1				
DMU 20	1.0127	λ_2=0.0728	λ_5=0.1482	λ_{10}=0.5144	λ_{19}=0.2646	
DMU 21	1	λ_{21}=1				
DMU 22	1	λ_{22}=1				

15 DMUs are found to be efficient whereas 7 DMUs are inefficient as portrayed by the table above. The output targets of the 7 inefficient DMUs are furnished below.

Table 7: Targets for inefficient DMUs

DMU	Name of the Banks	OP1	OP2	OP3
DMU 7	The Karnataka Bank Ltd.	9238.6017	18777.6275	60965.1248
DMU 8	The Karur Vysya Bank Ltd.	8039.2479	17033.8975	52429.8065
DMU 9	The Lakshmi Vitas Bank Ltd.	3601.0227	7578.9940	19726.1131
DMU 14	DCB Bank Ltd.	4741.7200	8373.1293	27413.2133
DMU 16	ICICI Bank Ltd.	98852.1916	265548.9630	686711.2814
DMU 18	Kotak Mahindra Bank Ltd.	44031.4998	90835.4888	265962.9847
DMU 20	Bandhan Bank	12944.8419	18491.8287	67476.4593

If the output is increased based on the above targets, efficiency could be attained. Out of the 15 efficient DMUs, 4 DMUs are categorized as weak efficient. The ranking of the remaining 11 efficient DMUs are given below.

Table 8: Ranking of DMUs

DMU	Counts	Rank
DMU 1	3	4
DMU 2	1	6
DMU 3	1	6
DMU 4	2	5
DMU 5	2	5
DMU 10	5	2
DMU 12	3	4
DMU 15	4	3
DMU 17	1	6
DMU 19	7	1
DMU 21	1	6

The table indicates that DMU19, DMU10, DMU 15 are ranked 1, 2, 3 respectively It may also be observed that tie exist in this ranking such as the tie for Rank 6 exhibited by four DMUs, namely DMU2, DMU3, DMU17 and DMU 21.

VRS Super-Efficiency Scores

Applying Model V to Private Sector Banks data, the super-efficiency score and peer weights for each DMU are obtained and listed below.

Table 9: Super-Efficiency Scores and Weights – VRS SE DEA Model

DMU	Efficiency Score	Weights for Reference DMU				
DMU 1	Infeasible					
DMU 2	0.8996	λ_5 =0.0997	λ_{10} =0.8827	λ_{15} =0.0077	λ_{19}=0.0098	
DMU 3	0.9203	λ_4 =0.5320	λ_{10}=0.4624	λ_{15}=0.0056		
DMU 4	0.8957	λ_{10}=0.9722	λ_{15}=0.0003	λ_{21} =0.0193	λ_{22}=0.0082	
DMU 5	0.8357	λ_{10}=0.9007	λ_{15}=0.0994			
DMU 6	Infeasible					
DMU 7	1.0702	λ_1 =0.0794	λ_{10} =0.1225	λ_{12}=0.7386	λ_{19}=0.0595	
DMU 8	1.0807	λ_1 =0.4989	λ_{10} =0.4106	λ_{15} =0.0200	λ_{17}=0.0532	λ_{19}=0.0173
DMU 9	1.4077	λ_3 =0.1191	λ_4 =0.7314	λ_{10}=0.0964	λ_{15}=0.0050	λ_{19}=0.0481
DMU 10	Infeasibl3					
DMU 11	0.9593	λ_{10}=0.8093	λ_{15}=0.0219	λ_{19} =0.1674	λ_{22}=0.0013	
DMU 12	0.9004	λ_1 =0.3212	λ_2 =0.4914	λ_5=0.1704	λ_{15}=0.0119	λ_{19}=0.0051
DMU 13	0.9780	λ_1=0.0939	λ_{12}=0.2947	λ_{15}=0.5253	λ_{19}=0.0861	
DMU 14	1.0816	λ_1=0.0429	λ_{12}=0.6921	λ_{12}=0.2068	λ_{19}=0.0582	
DMU 15	0.3889	λ_6 =0.0325	λ_{13}=0.9675			
DMU 16	1.0642	λ_4=0.0983	λ_{15}=0.6483	λ_{19} =0.2346	λ_{21}=0.0188	
DMU 17	0.8407	λ_1=0.6390	λ_{15}=0.1323	λ_{19} =0.2286		
DMU 18	1.2103	λ_5=0.0453	λ_{12}=0.2888	λ_{15} =0.1552	λ_{19}=0.5107	
DMU 19	0.5395	λ_{11} =0.2508	λ_{15} =0.0170	λ_{17} =0.1803	λ_{20}=0.1271	λ_{21}=0.4249
DMU 20	1.0127	λ_2=0.0728	λ_5=0.1482	λ_{10} =0.5144	λ_{19}=0.2646	
DMU 21	0.5896	λ_4=0.4262	λ_{19}=0.5738			
DMU 22	0.7095	λ_{10}=0.8556	λ_{15}=0.1444			

From the above table, it may be observed that infeasible solution exists for DMU1, DMU6 and DMU10. It is also observed from the above table that there are 15 extreme efficient DMUs ($\phi^{SE*} < 1$ and includes 3 infeasible DMUs) and 7 inefficient DMUs ($\phi^{SE*} > 1$). Also, it is identified there is no efficient and weak efficient DMUs. The following table gives the new facet target for each extreme efficient DMU in respect of their reference set.

Table 10: VRS New Facet Target

DMU	OP1	OP2	OP3
DMU 2	3591.8043	8517.1049	24933.9699
DMU 3	1696.6640	4932.6921	10802.2425
DMU 4	1297.5142	3298.4877	6752.0241
DMU 5	14373.4667	40439.0116	102176.3653
DMU 11	10000.0277	17410.4887	53768.6841
DMU 12	7931.7933	18570.2399	58018.9174
DMU 13	78839.5468	216522.8874	558880.6731
DMU 15	75925.0392	152392.8688	554958.0322
DMU 17	30040.7536	67719.2731	192380.8621
DMU 19	20459.0068	43274.0950	112581.7562
DMU 21	22230.3842	26768.9722	101147.7335
DMU 22	20562.3151	58019.3249	146780.4246

The facet targets exhibits efficient stability region and to prove this, DMU2 is considered, and is presented in the following table.

Table 11: DMU2 Reference Points

Variables		Original Values	Reference Point
IP1	Deposits	36825.03	36825.03
IP2	Fixed Assets	128.46	128.46
IP3	Operating Expenses	850.91	850.91
IP4	Staff	4325	4325
IP5	Paid-up Capital	142.51	142.51
OP1	Total income	3992.53	3591.80
OP2	Investments	9467.33	8517.10
OP3	Loan & Advances	27715.8	24933.97
	ϕ^{SE}	0.8996	0.9999
	ϕ	1	1

It may be noted that the super-efficiency score of DMU2 remains extreme efficient if all the outputs of DMU2 are scaled down 0.8996 times proportionally with inputs unchanged. Similarly, all extreme efficient DMUs are interpreted.

Condition for Infeasibility

Seiford and Zhu (1999) stated a necessary and sufficient condition for infeasibility through which the existence of infeasibility among the DMUs considered in this study has been verified and furnished below.

Table 12: Condition for Infeasibility

DMU	Name of the Banks	h*(infeasible test)
DMU 1	City Union Bank Ltd.	1.0423 ($h^* > 1$)
DMU 2	Tamilnad Mercantile Bank Ltd.	0.5402
DMU 3	The Catholic Syrian Bank Ltd.	0.4863
DMU 4	Dhanlaxmi Bank Ltd	0.7043
DMU 5	The Federal Bank Ltd.	0.1921
DMU 6	The Jammu & Kashmir Bank Ltd.	1.0332 ($h^* > 1$)
DMU 7	The Karnataka Bank Ltd.	0.2464
DMU 8	The Karur Vysya Bank Ltd.	0.4711
DMU 9	The Lakshmi Vitas Bank Ltd.	0.3581
DMU 10	The National Bank Ltd.	3.9392 ($h^* > 1$)
DMU 11	RBL Bank	0.1522
DMU 12	The South Indian Bank Ltd.	0.4169
DMU 13	Axis Bank Ltd.	0.1299

DMU 14	DCB Bank Ltd.	0.2529
DMU 15	HDFC Bank Ltd.	0.1336
DMU 16	ICICI Bank Ltd.	0.0570
DMU 17	Indusind Bank Ltd.	0.1094
DMU 18	Kotak Mahindra Bank Ltd.	0.0527
DMU 19	Yes Bank Ltd.	0.0729
DMU 20	Bandhan Bank	0.1345
DMU 21	IDFC First Bank	0.1179
DMU 22	IDBI Bank Ltd.	0.0548

From the above table, it may be observed that the DMUs which are identified as infeasible under VRS SE Model satisfied the necessary and sufficient condition for infeasibility ($h^* > 1$).

Sensitivity Analysis

Sensitivity analysis is generally considered to be an important part of DEA; hence, an attempt has been made to do sensitivity analysis in the SE DEA model, specifically VRS kth particular output models. The following section presents the calculated SE scores for the VRS kth specific output DEA model.

VRS kth Specfic Output DEA Model

Next, the author attempted VRS kth specific output DEA Model. The results for the various combinations of k are calculated and presented below.

Table 13: VRS kth Specific Output SE Scores

DMU	$\Phi^{SE} = \alpha^0_{k=\{1,2,3\}}$	$\alpha^0_{k=\{1\}}$	$\alpha^0_{k=\{2\}}$	$\alpha^0_{k=\{3\}}$	$\alpha^0_{k=\{1,2\}}$	$\alpha^0_{k=\{1,3\}}$	$\alpha^0_{k=\{2,3\}}$
DMU 1	Infeasible	Infeasible	Infeasible	Infeasible	Infeasible	Infeasible	Infeasible
DMU 2	0.8996	Infeasible	Infeasible	Infeasible	Infeasible	Infeasible	Infeasible
DMU 3	0.9203	Infeasible	0.9193	Infeasible	0.9193	Infeasible	0.9201
DMU 4	0.8957	Infeasible	0.8957	Infeasible	0.8957	Infeasible	0.8957
DMU 5	0.8357	Infeasible	Infeasible	0.8136	Infeasible	0.8357	0.8136
DMU 6	Infeasible	Infeasible	Infeasible	Infeasible	Infeasible	Infeasible	Infeasible
DMU 7	1.0702	1.1783	1.2606	1.0708	1.1344	1.0708	1.0702
DMU 8	1.0807	1.1346	1.2527	1.0891	1.1181	1.0890	1.0807
DMU 9	1.4077	1.6841	1.4968	1.5725	1.4079	1.5725	1.4143
DMU 10	Infeasible	Infeasible	Infeasible	Infeasible	Infeasible	Infeasible	Infeasible
DMU 11	0.9593	0.9401	Infeasible	Infeasible	0.9593	0.9401	Infeasible
DMU 12	0.9004	Infeasible	Infeasible	Infeasible	Infeasible	0.8716	Infeasible
DMU 13	0.9780	Infeasible	Infeasible	0.9780	Infeasible	0.9780	0.9780
DMU 14	1.0816	1.2262	1.3117	1.0842	1.1628	1.0842	1.0816
DMU 15	0.3889	Infeasible	Infeasible	Infeasible	Infeasible	Infeasible	Infeasible
DMU 16	1.0642	1.0772	1.0725	1.0712	1.0698	1.0712	1.0642
DMU 17	0.8407	Infeasible	Infeasible	Infeasible	Infeasible	0.8407	Infeasible
DMU 18	1.2103	1.3672	1.2758	1.2207	1.2602	1.2207	1.2103
DMU 19	0.5395	Infeasible	Infeasible	Infeasible	Infeasible	0.5352	Infeasible
DMU 20	1.0127	1.0502	1.3030	1.0127	1.0502	1.0127	1.0127
DMU 21	0.5896	Infeasible	0.5896	Infeasible	0.5896	Infeasible	0.5896
DMU 22	0.7095	Infeasible	0.5605	Infeasible	0.7095	Infeasible	0.5605

The above table indicate that VRS kth specific output model reflects feasibility and infeasibility in respect of various combinations of k. A few observations are noted here.

i. DMU1, DMU2, DMU6, DMU10 and DMU15 are infeasible in respect of all the kth specific output models.

ii. DMU17 and DMU19 are feasible with respect to the SE Model and kth specific model $\alpha^0_{k=\{1,3\}}$.

iii. DMU3, DMU4, DMU21 and DMU22 are feasible with respect to the models $\alpha^0_{k=\{2\}}$, $\alpha^0_{k=\{1,2\}}$ and $\alpha^0_{k=\{2,3\}}$

Further, the construction of the stability region for extreme efficient DMUs is attempted and presented below.

Table 14: VRS Output Stability Region

DMU	$\alpha^0_{k=\{1,2,3\}}$	$\alpha^0_{k=\{1\}}$	$\alpha^0_{k=\{2\}}$	$\alpha^0_{k=\{3\}}$	$\alpha^0_{k=\{1,2\}}$	$\alpha^0_{k=\{1,3\}}$	$\alpha^0_{k=\{2,3\}}$
DMU 1	$\alpha_{1,2,3} \leq 1$	$\alpha_1 \leq 1$	$\alpha_2 \leq 1$	$\alpha_3 \leq 1$	$\alpha_{1,2} \leq 1$	$\alpha_{1,3} \leq 1$	$\alpha_{2,3} \leq 1$
DMU 2	$0.8996 < \alpha_{1,2,3} \leq 1$	$\alpha_1 \leq 1$	$\alpha_2 \leq 1$	$\alpha_3 \leq 1$	$\alpha_{1,2} \leq 1$	$\alpha_{1,3} \leq 1$	$\alpha_{2,3} \leq 1$
DMU 3	$0.9203 < \alpha_{1,2,3} \leq 1$	$\alpha_1 \leq 1$	$0.9193 < \alpha_2 \leq 1$	$\alpha_3 \leq 1$	$0.9193 < \alpha_{1,2} \leq 1$	$\alpha_{1,3} \leq 1$	$0.9201 < \alpha_{2,3} \leq 1$
DMU 4	$0.8957 < \alpha_{1,2,3} \leq 1$	$\alpha_1 \leq 1$	$0.8957 < \alpha_2 \leq 1$	$\alpha_3 \leq 1$	$0.8957 < \alpha_{1,2} \leq 1$	$\alpha_{1,3} \leq 1$	$0.8957 < \alpha_{2,3} \leq 1$
DMU 5	$0.8357 < \alpha_{1,2,3} \leq 1$	$\alpha_1 \leq 1$	$\alpha_2 \leq 1$	$0.8136 < \alpha_3 \leq 1$	$\alpha_{1,2} \leq 1$	$0.8357 < \alpha_{1,3} \leq 1$	$0.8136 < \alpha_{2,3} \leq 1$
DMU 6	$\alpha_{1,2,3} \leq 1$	$\alpha_1 \leq 1$	$\alpha_2 \leq 1$	$\alpha_3 \leq 1$	$\alpha_{1,2} \leq 1$	$\alpha_{1,3} \leq 1$	$\alpha_{2,3} \leq 1$
DMU 10	$\alpha_{1,2,3} \leq 1$	$\alpha_1 \leq 1$	$\alpha_2 \leq 1$	$\alpha_3 \leq 1$	$\alpha_{1,2} \leq 1$	$\alpha_{1,3} \leq 1$	$\alpha_{2,3} \leq 1$
DMU 11	$0.9593 < \alpha_{1,2,3} \leq 1$	$0.9401 < \alpha_1 \leq 1$	$\alpha_2 \leq 1$	$\alpha_3 \leq 1$	$0.9593 < \alpha_{1,2} \leq 1$	$0.9401 < \alpha_{1,3} \leq 1$	$\alpha_{2,3} \leq 1$
DMU 12	$0.9004 < \alpha_{1,2,3} \leq 1$	$\alpha_1 \leq 1$	$\alpha_2 \leq 1$	$\alpha_3 \leq 1$	$\alpha_{1,2} \leq 1$	$0.8716 < \alpha_{1,3} \leq 1$	$\alpha_{2,3} \leq 1$
DMU 13	$0.9780 < \alpha_{1,2,3} \leq 1$	$\alpha_1 \leq 1$	$\alpha_2 \leq 1$	$0.9780 < \alpha_3 \leq 1$	$\alpha_{1,2} \leq 1$	$0.9780 < \alpha_{1,3} \leq 1$	$0.9780 < \alpha_{2,3} \leq 1$
DMU 15	$0.3889 < \alpha_{1,2,3} \leq 1$	$\alpha_1 \leq 1$	$\alpha_2 \leq 1$	$\alpha_3 \leq 1$	$\alpha_{1,2} \leq 1$	$\alpha_{1,3} \leq 1$	$\alpha_{2,3} \leq 1$
DMU 17	$0.8407 < \alpha_{1,2,3} \leq 1$	$\alpha_1 \leq 1$	$\alpha_2 \leq 1$	$\alpha_3 \leq 1$	$\alpha_{1,2} \leq 1$	$0.8407 < \alpha_{1,3} \leq 1$	$\alpha_{2,3} \leq 1$
DMU 19	$0.5395 < \alpha_{1,2,3} \leq 1$	$\alpha_1 \leq 1$	$\alpha_2 \leq 1$	$\alpha_3 \leq 1$	$\alpha_{1,2} \leq 1$	$0.5352 < \alpha_{1,3} \leq 1$	$\alpha_{2,3} \leq 1$
DMU 21	$0.5896 < \alpha_{1,2,3} \leq 1$	$\alpha_1 \leq 1$	$0.5896 < \alpha_2 \leq 1$	$\alpha_3 \leq 1$	$0.5896 < \alpha_{1,2} \leq 1$	$\alpha_{1,3} \leq 1$	$0.5896 < \alpha_{2,3} \leq 1$
DMU 22	$0.7095 < \alpha_{1,2,3} \leq 1$	$\alpha_1 \leq 1$	$0.5605 < \alpha_2 \leq 1$	$\alpha_3 \leq 1$	$0.7095 < \alpha_{1,2} \leq 1$	$\alpha_{1,3} \leq 1$	$0.5605 < \alpha_{2,3} \leq 1$

Next, sensitivity analysis for DMU4 based on the stability region constructed above is displayed below.

Table 15: Sensitivity analysis for DMU4

Variable	Original Value	$\alpha^0_{k=\{1\}}$	$\alpha^0_{k=\{2\}}$	$\alpha^0_{k=\{3\}}$	$\alpha^0_{k=\{1,2\}}$	$\alpha^0_{k=\{1,3\}}$	$\alpha^0_{k=\{2,3\}}$
IP1	10904.07	10904.07	10904.07	10904.07	10904.07	10904.07	10904.07
IP2	213.7	213.7	213.7	213.7	213.7	213.7	213.7
IP3	324.06	324.06	324.06	324.06	324.06	324.06	324.06
IP4	1714	1714	1714	1714	1714	1714	1714
IP5	253.01	253.01	253.01	253.01	253.01	253.01	253.01
OP1	1100.44	1100.44X 0.41	1100.44	1100.44	1100.44X 0.8959	1100.44X 0.01	1100.44
OP2	3682.4	3682.4	3682.4 X 0.8959	3682.4	3682.4 X 0.8959	3682.4	3682.4 X 0.8959
OP3	6496.1	6496.1	6496.1	6496.1X 0.01	6496.1	6496.1X 0.01	6496.1 X 0.8959
Standard CRS model	1	1	1	1	1	1	1
CRS SE	ϕ^{SE} = 0.8957	ϕ^{SE} = 0.8957	ϕ^{SE} = 0.9998	ϕ^{SE} = 0.8957	ϕ^{SE} = 0.9998	ϕ^{SE} = 0.8957	ϕ^{SE} = 0.9998
Output Specific Model	$\alpha^0_{k=\{1,2,3\}}$ =0.8957	$\alpha^0_{k=\{1\}}$= Infeasible	$\alpha^0_{k=\{2\}}$ = 0.9998	$\alpha^0_{k=\{3\}}$= Infeasible	$\alpha^0_{k=\{1,2\}}$= 0.9998	$\alpha^0_{k=\{1,3\}}$= Infeasible	$\alpha^0_{k=\{2,3\}}$= 0.9998

It may be observed that DMU4 is efficient in BCC Model, extreme efficient in VRS SE Model and feasible or infeasible in specific output BCC models for different combinations of k, before and after data variations. Thus OSR maintains the stability in efficiency classification.

Conclusion

The author's primary focus in this paper is the sensitivity analysis of super-efficiency DEA Models. For the purpose of empirical research, data on Private Sector Banks that are active in India through 2019–20 period were taken into consideration. There are 22 Private Sector Banks in the data structure. Every bank is regarded as a decision-making unit. Three outputs and five inputs are taken into account. To assess the technical efficiency and ranking of DMUs, the author first used the conventional DEA model and the super-efficiency DEA model. Furthermore, a sensitivity analysis is performed on the DEA results. The following is a summary and presentation of the findings from the use of different DEA models.

The use of output-oriented BCC shows that thirteen banks were operating profitably. There is a tie in the ranking of DMUs under the BCC Model because Indian Bank, Oriental Bank of Commerce, Union Bank of India, and State Bank of India all receive Rank 1 under the BCC Model. The super-efficiency DEA model is used since there are ties in the DMU ranking. Thirteen extremely efficient DMUs and five inefficient DMUs are represented in the VRS SE DEA model. It should be noted that two of the thirteen extremely efficient DMUs—Indian Bank and Punjab & Sind Bank—have an infeasible solution, which is confirmed by Zhu's (2003) infeasibility test.

VRS kth specific output model highlights some of the results are feasible and infeasible and the following observations are noted. City Union Bank Ltd., Tamilnad Mercantile Bank Ltd., The Jammu & Kashmir Ltd., The National Bank Ltd. and HDFC Bank Ltd. are infeasible in respect of all the kth specific output models. Indusind Bank Ltd. and Yes Bank Ltd. are feasible with respect to the SE Model and kth specific model $\alpha^0_{k=\{1,3\}}$. The Catholic Syrian Bank Ltd., Dhanlaxmi Bank Ltd., IDFC First Bank Ltd. and IDBI Bank Ltd. are feasible with respect to the models $\alpha^o_{k=\{2\}}, \alpha^o_{k=\{1,2\}}$ and $\alpha^o_{k=\{2,3\}}$. Sensitivity Analysis done on Dhanlaxmi Bank Ltd. showed that the bank is efficient in BCC Model, extreme efficient in VRS SE Model and feasible or infeasible in specific output BCC models for different combinations of k, before and after data variations. Thus OSR maintains the stability in efficiency classification.

Sensitivity analysis lends credibility to any type of financial model by testing it under many scenarios. Financial sensitivity analysis provides the analyst with a great deal of flexibility when determining the parameters for assessing the sensitivity of the dependent variables to the independent factors. Making decisions is facilitated by sensitivity analysis. Decision-makers use the model to ascertain how sensitive the outcome is to changes in specific factors. Because of this, the analysis could be useful in reaching certain conclusions and be vital in assisting the banking industry in making the best decisions.

Future scope of research

DEA may be performed in respect of Foreign Sector Banks in India. Sensitivity Analysis relating to data perturbations in both input and output could be carried out. The model due to Cook et al (2009) relating to overcoming the problem of infeasibility may also be attempted. Researchers can look into the possibility of improving and modifying current DEA methodology available in DEA literature.

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